Using Gauss-Jordan elimination (pivot method) as shown in lecture & website handout, solve the following systems of linear equations. **For each question, you must produce a matrix in reduced row echelon form (RREF) as part of your work. Write your final solutions in co-ordinate form. You do NOT need to check your answers.**

SCORE: _____ / 14 PTS

[a]

$$\begin{aligned} 3x - 7y - 2z &= 7 \\ -2x + 3y - 2z &= 0 \\ -x + 3y + 2z &= -5 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & -7 & -2 & 7 \\ -2 & 3 & -2 & 0 \\ -1 & 3 & 2 & -5 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} -1 & 3 & 2 & -5 \\ -2 & 3 & -2 & 0 \\ 3 & -7 & -2 & 7 \end{array}\right] -R_1$$

1

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 5 \\ -2 & 3 & -2 & 0 \\ 3 & -7 & -2 & 7 \end{array}\right] \begin{matrix} R_2+2R_1 \\ R_3+(-3)R_1 \end{matrix}$$

2

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 5 \\ 0 & -3 & -6 & 10 \\ 0 & 2 & 4 & -8 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 5 \\ 0 & 2 & 4 & -8 \\ 0 & -3 & -6 & 10 \end{array}\right] \tfrac{1}{2}R_2$$

1

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 5 \\ 0 & 1 & 2 & -4 \\ 0 & -3 & -6 & 10 \end{array}\right] R_3+3R_2$$

1

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 5 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

NO SOLUTION

1

[b]

$$\begin{aligned} x + 2y - 2z - 3w &= 2 \\ -2x - 3y + 4z + 10w &= -10 \\ 4x + 6y - 8z - 19w &= 19 \\ x + 3y - 2z + 2w &= -5 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -3 & 2 \\ -2 & -3 & 4 & 10 & -10 \\ 4 & 6 & -8 & -19 & 19 \\ 1 & 3 & -2 & 2 & -5 \end{array}\right] \begin{matrix} \\ R_2+2R_1 \\ R_3+(-4)R_1 \\ R_4+(-1)R_1 \end{matrix}$$

3

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -3 & 2 \\ 0 & 1 & 0 & 4 & -6 \\ 0 & -2 & 0 & -7 & 11 \\ 0 & 1 & 0 & 5 & -7 \end{array}\right] \begin{matrix} \\ \\ R_3+2R_2 \\ R_4+(-1)R_2 \end{matrix}$$

$1\frac{1}{2}$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -3 & 2 \\ 0 & 1 & 0 & 4 & -6 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] R_4+(-1)R_3$$

$\frac{1}{2}$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -3 & 2 \\ 0 & 1 & 0 & 4 & -6 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} R_1+3R_3 \\ R_2+(-4)R_3 \\ \\ \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & -1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_1+(-2)R_2$$

$1\frac{1}{2}$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} x-2z=3 \rightarrow x=2z+3 \\ y=-2 \\ w=-1 \\ \end{matrix}$$

$1\frac{1}{2}$ $(2z+3, -2, z, -1)$

Using Gauss-Jordan elimination (pivot method) as shown in lecture & website handout, solve the following problems. **For each problem, you must produce a matrix in reduced row echelon form (RREF) as part of your work.** **SCORE: _____ / 16 PTS**

Identify upfront what each unknown represents
Scale your original equations so all coefficients are integers before you write the matrix
Summarize your answer in sentence form (including units of measurement)

[a] A tip jar contains a mixture $1, $2 and $5 bills, with four more $2 bills than $5 bills. If the jar contains 53 bills totaling $87, how many of each type of bill are in the jar ?

(2)
O = # OF $1 BILLS
t = $2
f = $5

(3)
$o + t + f = 53$
$o + 2t + 5f = 87$
$t - f = 4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 53 \\ 1 & 2 & 5 & 87 \\ 0 & 1 & -1 & 4 \end{array}\right] R_2 + (-1)R_1$$

(1)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 53 \\ 0 & 1 & 4 & 34 \\ 0 & 1 & -1 & 4 \end{array}\right] R_3 + (-1)R_2$$

(1)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 53 \\ 0 & 1 & 4 & 34 \\ 0 & 0 & -5 & -30 \end{array}\right] -\tfrac{1}{5}R_2$$

($\frac{1}{2}$)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 53 \\ 0 & 1 & 4 & 34 \\ 0 & 0 & 1 & 6 \end{array}\right] \begin{array}{l} R_1 + (-1)R_3 \\ R_2 + (-4)R_3 \end{array}$$

(1)
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 47 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 6 \end{array}\right] R_1 + (-1)R_2$$

($\frac{1}{2}$)
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 37 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

THE JAR HAS 37 $1 BILLS
10 $2 BILLS
6 $5 BILLS
(1)

[b] A mixture of 9 pounds of fertilizer A, 5 pounds of fertilizer B, and 5 pounds of fertilizer C provides the optimal nutrients for a plant.
Commercial brand X contains equal parts of fertilizer A and fertilizer C.
Commercial brand Y contains two parts of fertilizer A and one part of fertilizer B.
Commercial brand Z contains equal parts of fertilizer A, fertilizer B and fertilizer C.
How many pounds of each commercial brand are needed to obtain the desired mixture of fertilizer ?

(2)
x = # POUNDS OF BRAND X
y = Y
z = Z

EACH POUND OF CONTAINS
X: $\frac{1}{2}$ LB A , $\frac{1}{2}$ LB C
Y: $\frac{2}{3}$ LB A , $\frac{1}{3}$ LB B
Z: $\frac{1}{3}$ LB A , $\frac{1}{3}$ LB B , $\frac{1}{3}$ LB C

(3)
$\frac{1}{2}x + \frac{2}{3}y + \frac{1}{3}z = 9$
$\frac{1}{3}y + \frac{1}{3}z = 5$
$\frac{1}{2}x + \frac{1}{3}z = 5$

(1)
$3x + 4y + 2z = 54$
$y + z = 15$
$3x + 2z = 30$

$$\left[\begin{array}{ccc|c} 3 & 4 & 2 & 54 \\ 0 & 1 & 1 & 15 \\ 3 & 0 & 2 & 30 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 2 & 30 \\ 0 & 1 & 1 & 15 \\ 3 & 4 & 2 & 54 \end{array}\right] \tfrac{1}{3}R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 10 \\ 0 & 1 & 1 & 15 \\ 3 & 4 & 2 & 54 \end{array}\right] R_3 + (-3)R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 10 \\ 0 & 1 & 1 & 15 \\ 0 & 4 & 0 & 24 \end{array}\right] R_3 + (-4)R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 10 \\ 0 & 1 & 1 & 15 \\ 0 & 0 & -4 & -36 \end{array}\right] -\tfrac{1}{4}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 10 \\ 0 & 1 & 1 & 15 \\ 0 & 0 & 1 & 9 \end{array}\right] \begin{array}{l} R_1 + (-\frac{2}{3})R_3 \\ R_2 + (-1)R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

4 LBS OF X,
6 LBS OF Y AND
9 LBS OF Z
ARE NEEDED